

Math 0372
Week 11 - (Apr. 8 - Apr. 12)
Spring 2019

Note to the instructors: These problems are based on the report generated by ALEKS.
This homework contains composition of functions and inverse functions.
Let me know if you have any suggestions.

-Melissa

MATH 0372

Foundations of Pre-Calculus: Week 11 - (Apr. 8 - Apr. 12)

Note to the students: Show all work on a separate sheet of paper. We almost never want "just an answer". We like for you to show that you know the concept that was taught and that you can justify your answer.

Worksheet #18 - Composition of Functions.

* The Composition of Functions is used when determining whether two functions are inverses of each other, so it's necessary to know how to compose two functions. The notation is sometimes hard to Understand for some. The composition of f and g , denoted $f \circ g$, is defined by, $(f \circ g)(x) = f(g(x))$. The domain of $f \circ g$ is the set of real numbers x in the domain of g such that $g(x)$ is in the domain of f . (This should be a review from several weeks ago.) Let's practice.

* Example: Given $f(x) = 2x + 2$ and $g(x) = -2x^2 - 1$, find $(g \circ f)(-5)$.

Solution: Recall the definition above for composition of functions? We will re-write this to make it easier to understand. (Sometimes getting past the math notation is the biggest part for some students.)

So, $(g \circ f)(-5) = g(f(-5))$. We need to first find $f(-5)$.

$f(-5) = 2(-5) + 2 = -8$. Now substitute $f(-5) = -8$ and we find $g(f(-5)) = g(-8)$.

Now find $g(-8) = -2(-8)^2 - 1 = -2(64) - 1 = -129$

This means, $(g \circ f)(-5) = -129$

Use the functions f , g , and h to work the problems below:

$$f(x) = x^3 - 4x \qquad g(x) = \sqrt{2x} \qquad h(x) = 2x + 3$$

1. Find $f(g(8))$
2. Find $h(f(1))$
3. Find $(f \circ g)(18)$
4. Find $(h \circ f)(-3)$
5. Find $(g \circ f)(1)$
6. Find $(f \circ f)(3)$
7. Find $(h \circ h)(-4)$
8. Find $(h \circ h)(x)$

Use the functions m , n , p and q to work the problems below. Find the indicated function and write the domain using interval notation.

$$m(x) = \sqrt{x+8} \qquad n(x) = x - 5 \qquad p(x) = x^2 - 9x \qquad q(x) = \frac{1}{x-10}$$

* Example: Find $(n \circ p)(x)$. So, this means find $n(p(x))$. Since $p(x) = x^2 - 9x$, we find $n(x^2 - 9x) = (x^2 - 9x) - 5$. There are no like terms, so we can just write $(n \circ p)(x) = x^2 - 9x - 5$. There was no restriction on the domain for $p(x)$, so the domain for $(n \circ p)(x) = x^2 - 9x - 5$ is $(-\infty, \infty)$.

9. Find $(p \circ n)(x)$
10. Find $(m \circ n)(x)$
11. Find $(n \circ m)(x)$
12. Find $(q \circ m)(x)$
13. Find $(q \circ q)(x)$

Worksheet #19 - Inverse Functions.

- * Definition of an Inverse Function: Let f be a one-to-one function. Then g is the inverse of f if the following conditions are true.

- (a) $(f \circ g)(x) = x$ for all x in the domain of g .
- (b) $(g \circ f)(x) = x$ for all x in the domain of f .

- * Reminder: Definition of a One-to-One Function.

A function f is a one-to-one function, if for a and b in the domain of f , if $a \neq b$, then $f(a) \neq f(b)$, or equivalently, if $f(a) = f(b)$, then $a = b$.

- * To find an Equation of an Inverse of a Function:

For a one-to-one function defined by $y = f(x)$, the equation of the inverse can be found as follows.

- (a) Replace $f(x)$ by y .
- (b) Interchange x and y .
- (c) Solve for y .
- (d) Replace y by $f^{-1}(x)$.

- * Example: Determine whether the two functions are inverses. $f(x) = 5x + 4$ and $g(x) = \frac{x-4}{5}$.

First we find $f(g(x)) = 5(\frac{x-4}{5}) + 4$. The five cancels and we get $f(g(x)) = (x - 4) + 4$.

So, $f(g(x)) = x$. Now we need to find $g(f(x)) = \frac{(5x+4)-4}{5}$. When we simplify the numerator, we get, $g(f(x)) = \frac{5x}{5} = x$. Since both $f(g(x)) = x$ and $g(f(x)) = x$ for all x in the domain, these are inverses.

- * For the following problems, determine whether the two functions are inverses.

14. $m(x) = \frac{-2+x}{6}$ and $n(x) = 6x - 2$

15. $f(x) = \frac{4}{x-1}$ and $v(x) = \frac{x+4}{x}$

16. $p(x) = \frac{-3+x}{4}$ and $q(x) = 4x - 3$

17. $h(x) = 7x - 3$ and $k(x) = \frac{x+3}{7}$

* Example: Write an equation for the inverse function of the one-to-one function,
 $f(x) = \frac{4-x}{9}$.

Solution: First replace $f(x)$ with y , so we will write, $y = \frac{4-x}{9}$.

Next, interchange x and y , so we get, $x = \frac{4-y}{9}$.

Now, solve for y . Multiply both sides by 9 to get, $9x = 4 - y$.

Finish solving for y to get $y = 4 - 9x$.

The solution is $f^{-1}(x) = 4 - 9x$

For the following one-to-one functions, write an equation for the inverse function.

18. $g(x) = \frac{8-x}{3}$

19. $h(x) = \sqrt[3]{x-5}$

20. $m(x) = 4x^3 + 2$

21. $c(x) = \frac{5}{x+2}$

22. $v(x) = \frac{x-5}{x+1}$

23. $f(x) = \sqrt[5]{x-4} + 1$